

Nonsimilarity solutions for mixed convection from horizontal surfaces in a porous medium—variable surface heat flux

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Abstract—A nonsimilarity solution for mixed convection from impermeable horizontal surfaces in a saturated porous medium is obtained for the case of variable heat flux. Solutions that cover the entire regime of mixed convection, including the two limits of pure forced convection and pure free convection, are made possible through using two different transformations to the governing equations. The nonsimilarity parameter $\xi_f = Ra_f/Pe_f^2$ results from transformation of the governing equations for the forced flow dominated regime and the nonsimilarity parameter $\xi_n = Pe_n/Ra_n^{1/2}$ arises from transformation of the governing equations for the buoyancy dominated regime. The two solutions provide results that cover the entire mixed convection regime from pure forced to pure free convection limit. Numerical results for different values of surface heat variation are presented. Correlation equations for the local and average Nusselt numbers, valid for the entire mixed convection regime, are also presented.

INTRODUCTION

CONVECTIVE heat transfer for impermeable surfaces in a porous medium has many engineering applications in geothermal reservoirs and petroleum industries. The problem of mixed convective heat transfer from impermeable inclined surfaces in saturated porous media was studied by Cheng [1], who treated similarity solutions for wedge flows under the special cases in which the free-stream velocity and the wall temperature distribution vary according to the same power of the axial distance, x . For horizontal surfaces, condition was restricted to the case of constant heat flux, i.e. when the wall temperature varies according to $x^{1/2}$. Other simple geometries were also treated by Cheng [2] for mixed convective flows, but all were limited to the similarity and local similarity cases. A general similarity transformation for mixed convection flows in a porous medium was reported by Nakayama and Koyama [3] for different two-dimensional plane geometries and axisymmetric bodies of arbitrary shape.

Most of the studies to date have been based on Darcy's law. Darcy's model is considered valid when the flow is slow or when the pores of the porous medium are small [4]. The wall effect (non-slip condition) is more pronounced near the leading edge and decreases with increasing distance downstream, as discussed by

Vafai and Tien [5]. In general, the non-Darcian effect influences the velocity field more than the thermal field or the heat transfer rate [6]. Most of the published results, however, are limited to the cases where similarity [1, 2] or local similarity solution [3, 7] exists. The local similarity method provides numerical results that are of uncertain accuracy, especially for large values of the nonsimilarity parameter ξ where Darcy's model is valid. Also, there is no positive way to establish how the deleted terms involving the axial derivatives, the step needed to obtain local similarity, affect the final results [8]. Nonsimilar solutions for mixed convection about nonisothermal cylinders in cross flow and spheres in a porous medium are reported by Minkowycz *et al.* [9], where the local nonsimilarity solution method was employed and results were compared with those obtained from the local similarity solution method. Significant differences between the two solutions, 10–15%, were reported. Also Yucel [10] reported a non-similar solution that accounts for the effect of injection or suction of fluid on free convection about a vertical cylinder in a porous medium. Ranganathan and Viskanta [11] solved numerically the boundary layer equations for mixed convection along a vertical surface in a porous medium using a finite-difference scheme. Nakayama and Pop [7] presented a unified similarity transformation which is valid in the limits of pure forced

NOMENCLATURE

C_{fx}	local friction factor	Greek symbols	
f	dimensionless stream function	α	effective thermal diffusivity of saturated porous medium
$h(x)$	local heat transfer coefficient	β	volumetric coefficient of thermal expansion
\bar{h}	average heat transfer coefficient, $(1/L)\int_0^L h(x) dx$	δ	boundary layer thickness
k	thermal conductivity	η	pseudo-similarity variable
K	permeability coefficient of the porous medium	θ	dimensionless temperature
L	length of the plate	μ	dynamic viscosity
Nu_x	local Nusselt number, hx/k	ν	kinematic viscosity
\bar{Nu}	average Nusselt number, $\bar{h}L/k$	ζ_f	nonsimilarity parameter for the forced convection dominated regime
Pe_x	local Peclet number, $u_\infty x/\alpha$	ζ_n	nonsimilarity parameter for the free convection dominated regime
q_w	local surface heat flux	ρ	fluid density
Ra_x	modified local Rayleigh number, $g\beta q_w(x)Kx^2/(k\nu\alpha)$	ψ	stream function.
Re	Reynolds number, $u_\infty x/\nu$	Subscripts	
T	temperature	f	forced convection dominated condition
T_∞	free stream temperature	n	free convection dominated condition
T_w	wall temperature	x, y, ζ_f, ζ_n	partial derivatives with respect to x, y, ζ_f and ζ_n , respectively.
u, v	velocity components in x - and y -direction		
u_∞	free stream velocity		
x, y	axial and normal coordinates.		

convection and pure free convection. However, the cases they considered for solution were restricted to the local similarity approximation.

In the present work, nonsimilar solutions for mixed convection from a horizontal surface in a saturated porous medium are presented. To cover the entire mixed convection regime, two different transformations are used. In the first transformation the nonsimilarity parameter $\zeta_f = Ra_x/Pe_x^2$ is found to measure the buoyancy effect in the forced flow dominated mixed convection regime, and in the second transformation the nonsimilar parameter $\zeta_n = Pe_x/Ra_x^{1/2}$ is found to measure the forced flow effect in the buoyancy dominated mixed convection regime. The two solutions must match and overlap over the middle region to cover the entire mixed convection regime, from pure forced convection to pure free convection. Results are presented for different surface heat flux distributions.

ANALYSIS

Consider the combined free and forced convection in a porous medium adjacent to an impermeable, heated horizontal flat plate at the bottom. The axial and normal coordinates are x and y , and the corresponding velocity components are u and v , respectively. The gravitational acceleration g is acting downward in the direction opposite to the y coordinate.

The properties of the fluid and the porous medium are assumed to be constant and isotropic. In addition, the flow velocity and the pores of porous medium are assumed to be small for the Darcy model to be valid [4]. Under the Boussinesq and the boundary layer approximations, the governing equations are given by ref. [12]

$$u_x + v_y = 0 \quad (1)$$

$$\psi_{yy} = -(K\rho_\infty g\beta/\mu)T_x \quad (2)$$

$$T_{yy} = (1/\alpha)(\psi_y T_x - \psi_x T_y). \quad (3)$$

In the above equations, the stream function ψ satisfies the continuity equation with $u = \psi_y$ and $v = -\psi_x$, where u and v denote the Darcian velocity components in the x and y directions; T is the temperature; ρ , μ , and β are the density, viscosity, and the thermal expansion coefficient of the convecting fluid respectively; K is the permeability of the porous medium; and α is the equivalent thermal diffusivity of the porous medium. With power-law variation in the surface heat flux, the boundary conditions can be written as

$$\begin{aligned} y = 0: \quad q_w &= ax^n, v = 0 \\ y = \infty: \quad T &= T_\infty, u = u_\infty \end{aligned} \quad (4)$$

where a and n are prescribed constants. Note that $n = 0$ corresponds to the case of uniform surface heat flux.

Forced convection dominated case

In this case the following dimensionless variables are introduced

$$\eta = (y/x)Pe_x^{1/2}, \quad \xi_r = \xi_r(x) \quad (5)$$

$$\psi = \alpha Pe_x^{1/2} f(\xi_r, \eta),$$

$$\theta(\xi_r, \eta) = (T - T_\infty) Pe_x^{1/2} / [q_w(x)x/k]. \quad (6)$$

The governing equations and boundary conditions, equations (1)–(4), can then be transformed into

$$f'' + \xi_r[(n+1/2)\theta - (\eta/2)\theta'] = -n\xi_r^2\theta_{\xi_r} \quad (7)$$

$$\theta'' + (1/2)f\theta' - (n+1/2)f'\theta = n\xi_r(f'\theta_{\xi_r} - \theta'f_{\xi_r}) \quad (8)$$

$$f(\xi_r, 0) + 2n\xi_r f_{\xi_r}(\xi_r, 0) = 0 \quad \text{or} \quad f(\xi_r, 0) = 0$$

$$\theta'(\xi_r, 0) = -1, f'(\xi_r, \infty) = 1, \theta(\xi_r, \infty) = 0 \quad (9)$$

where

$$\xi_r(x) = Ra_x / Pe_x^2 \quad (10)$$

and the primes denote partial differentiations with respect to η .

In the above system of equations, the parameter ξ_r is a measure of the buoyancy effect on forced convection. The case of $\xi_r = 0$ corresponds to pure forced convection and the limiting case of $\xi_r = \infty$ corresponds to pure free convection. The solution of the system of equations (7)–(9) cannot be carried out to cover the entire regime of mixed convection because of the singularity at $\xi_r = \infty$. The above system of equations is used to generate results that apply to the forced convection dominating regime.

Some of the physical quantities of interest include the velocity components u and v in the x and y direction, the local friction factor C_{fx} (defined as $\tau_w/(\rho u_x^2)/2$, where $\tau_w = \mu(u_y)_{y=0}$), and the local Nusselt number $Nu_x = hx/k$, where $h = q_w/(T_w - T_\infty)$. They are given by

$$u = u_x f'(\xi_r, \eta) \quad (11)$$

$$v = -(\alpha/x)Pe_x^{1/2}[(1/2)f(\xi_r, \eta) - (\eta/2)f'(\xi_r, \eta) + n\xi_r f_{\xi_r}] \quad (12)$$

$$C_{fx} Pr^{-1} Pe_x^{1/2} = 2f''(\xi_r, 0) \quad (13)$$

and

$$Nu_x Pe_x^{-1/2} = 1/\theta(\xi_r, 0). \quad (14)$$

Free convection dominated case

For the buoyancy dominated case the following dimensionless variables are introduced in the transformation:

$$\eta = (y/x)Ra_x^{1/4}, \quad \xi_n = \xi_n(x) \quad (15)$$

$$\psi = \alpha Ra_x^{1/4} f(\xi_n, \eta),$$

$$\theta(\xi_n, \eta) = (T - T_\infty) Ra_x^{1/4} / [q_w(x)x/k]. \quad (16)$$

Substituting equations (15) and (16) into the governing equations (1)–(4) leads to

$$f'' + [(3n+2)/4]\theta + [(n-2)/4]\eta\theta' = (n/2)\xi_n\theta_{\xi_n} \quad (17)$$

$$\begin{aligned} \theta'' + [(n+2)/4]f\theta' - [(3n+2)/4]f'\theta \\ = -(n/2)\xi_n(f'\theta_{\xi_n} - \theta'f_{\xi_n}) \end{aligned} \quad (18)$$

$$(n+2)f(\xi_n, 0) - 2n\xi_n f_{\xi_n}(\xi_n, 0) = 0 \quad \text{or} \quad f(\xi_n, 0) = 0$$

$$\theta'(\xi_n, 0) = -1, f'(\xi_n, \infty) = \xi_n, \theta(\xi_n, \infty) = 0 \quad (19)$$

where

$$\xi_n(x) = Pe_x / Ra_x^{1/2} \quad (20)$$

and the primes in equations (17)–(19) again denote the partial differentiations with respect to η .

Note that the ξ_n parameter here is a measure of the forced flow effect on free convection. The case of $\xi_n = 0$ corresponds to pure free convection and the limiting case of $\xi_n = \infty$ corresponds to pure forced convection. The latter limit cannot be reached by solving the above system of equations (17)–(19) and these equations are used to generate results for the free convection dominating regime.

The velocity components u and v , the local friction factor, and the local Nusselt number for this case have the expressions

$$u = (\alpha/x)Ra_x^{1/2}f'(\xi_n, \eta) \quad (21)$$

$$v = -(\alpha/x)Ra_x^{1/4}\{[(n+2)/4]f(\xi_n, \eta) + [(n-2)/4]\eta f'(\xi_n, \eta) - (n/2)\xi_n f_{\xi_n}\} \quad (22)$$

$$C_{fx} Pe_x^2 Pr^{-1} Ra_x^{-3/4} = 2f''(\xi_n, 0) \quad (23)$$

and

$$Nu_x Ra_x^{-1/4} = 1/\theta(\xi_n, 0). \quad (24)$$

It is noted that the solution of the two systems of equations, equations (7)–(9) and (17)–(19), must match and overlap in the middle region of the mixed convection domain, thus forming the solution for the entire regime of mixed convection. Also, if the right-hand-side terms of equations (7), (8) and (17), (18) are set equal to zero (i.e. the derivative of the variables with respect to ξ is negligible or ξ is very small), then the respective system of equations reduces to the ‘local similarity’ model which has been treated by others (e.g. refs. [3, 7]). Note that the present nonsimilarity formulation reduces to similar boundary layer formulation for the special case of $n = 0$, which corresponds to the constant surface heat flux. Also, similar solutions for pure forced and pure free convection limits can be obtained from the above equations by setting $\xi_r = 0$ and $\xi_n = 0$ in equations (7)–(9) and (17)–(19), respectively.

The two systems of partial differential equations, equations (7)–(9) and (17)–(19), have the same general form which can be written as

$$f'' + m_1\theta + m_2\eta\theta' = m_3\theta_{\xi} \quad (25)$$

$$\theta'' + m_4f\theta' + m_5f'\theta = m_6(f'\theta_{\xi} - \theta'f_{\xi}) \quad (26)$$

with boundary conditions given by

$$\begin{aligned} f(\xi, 0) = 0, \quad \theta'(\xi, 0) = -1 \\ f'(\xi, \infty) = m_7, \quad \theta(\xi, \infty) = 0 \end{aligned} \quad (27)$$

where the coefficients m_1 to m_7 are generally functions of ξ . For the first system they are given by:

$$\begin{aligned} m_1 = \xi_r(n+1/2), \quad m_2 = -(1/2)\xi_r, \\ m_3 = -n\xi_r^2, \quad m_4 = 1/2, \quad m_5 = -(n+1/2), \\ m_6 = n\xi_r, \quad \text{and} \quad m_7 = 1. \end{aligned}$$

For the second system they are:

$$\begin{aligned} m_1 = (3n+2)/4, \quad m_2 = (n-2)/4, \quad m_3 = (n/2)\xi_n, \\ m_4 = (n+2)/4, \quad m_5 = -[(3n+2)/4], \\ m_6 = -(n/2)\xi_n, \quad \text{and} \quad m_7 = \xi_n. \end{aligned}$$

Each of the systems of equations was converted into a set of first order equations, which was then solved, along with the boundary conditions, by a finite-difference scheme due to Keller as described in Cebeci and Bradshaw [13]. To conserve space, the details of the solution procedure are omitted here.

RESULTS AND DISCUSSION

The range of n values for which the present problem is physically realistic can be found following the argument used by Cheng and Chang [12]. When the wall temperature at $x > 0$ is different from that of the surrounding, both u and δ , the streamwise velocity component and the boundary layer thickness, must increase or at least remain constant with respect to x . From equations (21) and (22) one finds that u varies like $x^{n/2}$ and v varies like $x^{(n-2)/4}$. Also, from equation (15) the boundary layer thickness δ , which is of the order of y , varies like $x^{(2-n)/4}$. Thus, the above conditions can be satisfied if $0 \leq n \leq 2$. Based on the above argument, the numerical computations were carried out for values of n within the above range. Results for $\theta(\xi_r, \eta)/\theta(\xi_r, 0)$ and $f'(\xi_r, \eta)$, the temperature and velocity profiles, for $0 \leq n < 2$ are presented in Figs. 1 and 2 for different values of ξ_r . These two figures show that for a given value of n , as the buoyancy parameter ξ_r increases the slip of the u -component velocity at the wall increases as a result of the buoyancy-induced favorable pressure gradient. Also, the gradient of the temperature profiles at the wall increases with ξ_r , which results in higher heat transfer rate. It is clear from Figs. 1 and 2 that the thermal and the momentum boundary layer thicknesses become smaller as ξ_r increases. The effect of the value of n , the exponent for the surface heat flux variation, on the velocity and temperature profiles can be clearly seen. As n increases, the thickness of the thermal boundary layer as well as the thickness of the momentum boundary layer decreases and the temperature gradient at the wall increases, which enhances the surface heat transfer rate.

From the relationship between ξ_n and ξ_r , $\xi_n = \xi_r^{-1/2}$, the local Nusselt number expression for the free

convection dominated regime, equation (24), can be expressed as

$$Nu_x Pe_x^{-1/2} = [1/\theta(\xi_n, 0)]\xi_r^{1/4}. \quad (28)$$

Thus, the Nusselt number results from solutions of the two systems of equations for different values of n can be presented in the form $Nu_x Pe_x^{-1/2}$ versus ξ_r for the entire mixed convection regime. This is illustrated in Fig. 3. The corresponding asymptotic values for pure forced and pure free convection are also presented in the figure. Figure 3 shows that the local Nusselt number increases as the value of n increases and as the buoyancy parameter ξ_r increases. The domains of pure forced convection, mixed convection, and pure free convection can be established from the present results based on a 5% departure in the local Nusselt number from the pure forced convection limit and from the pure free convection limit. They are listed in Table 1.

The values of $1/\theta(\xi_r, 0)$ and $1/\theta(\xi_n, 0)$ at selected values of ξ_r and ξ_n and for different values of n are listed in Table 2. Exact solutions for similar boundary layers exist for the cases of pure forced convection and pure free convection, as well as for the case of mixed convection with uniform surface heat flux ($n = 0$ case).

For practical purposes, correlation equations were developed for the local Nusselt numbers. By using the least square fitting technique the local Nusselt number for pure forced convection in the range of $0 \leq n \leq 2$ can be correlated by

$$Nu_f = f_1(n) Pe_x^{1/2} \quad (29)$$

where

$$f_1(n) = 0.8872 + 0.5298n - 0.1034n^2 + 0.0163n^3. \quad (30)$$

For the case of pure free convection, the corresponding correlation equation for the local Nusselt number is given by

$$Nu_n = f_2(n) Ra_x^{1/4} \quad (31)$$

where

$$f_2(n) = 0.8597 + 0.3596n - 0.0641n^2 + 0.0103n^3. \quad (32)$$

Equations (29) and (31) fit the computed results for pure forced and pure free convection within an error of less than 3%, respectively.

Following Churchill [14], the correlation equation for the local Nusselt number in mixed convection is expressed as

$$(Nu_x/Nu_f)^m = 1 + (Nu_n/Nu_f)^m. \quad (33)$$

For the present study the correlation equation for the local mixed convection Nusselt number can be presented by

$$\begin{aligned} Nu_x Pe_x^{-1/2} / f_1(n) \\ = [1 + \{f_2(n)(Ra_x/Pe_x^2)\}^{1/4} / f_1(n)]^{1/m}. \end{aligned} \quad (34)$$

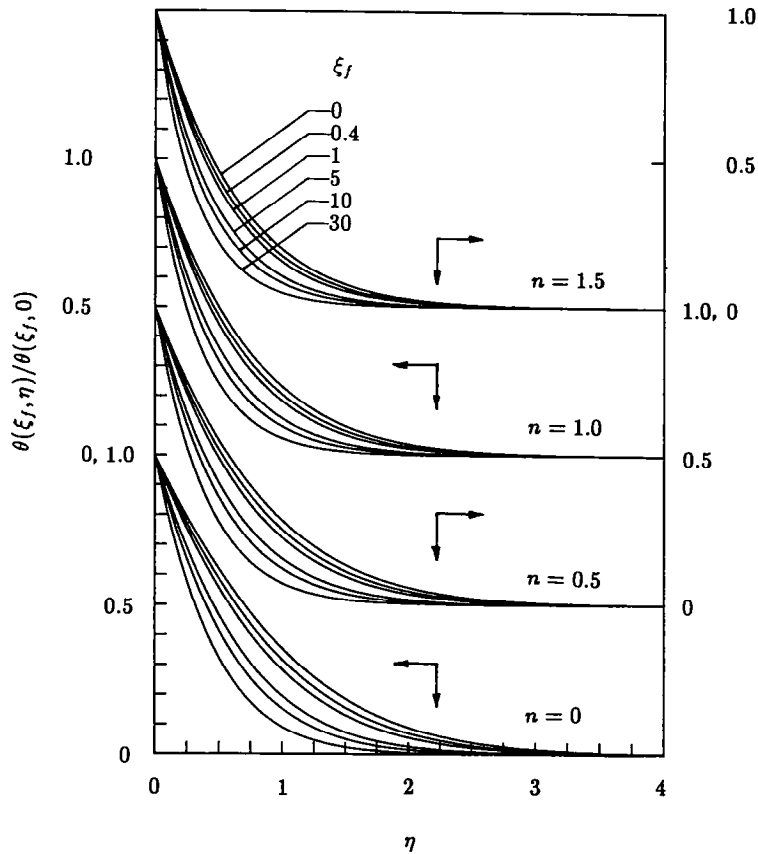


FIG. 1. Dimensionless temperature profiles at selected values of ξ_r and n .

The corresponding correlation equation for the average mixed convection Nusselt number $\overline{Nu} = \bar{h}L/k$, where \bar{h} is the average heat transfer coefficient over the plate length L , can be presented by

$$\overline{Nu}Pe_L^{-1/2}/2f_1(n) = [1 + \{[2/(n+2)]f_2(n)(Ra_L/Pe_L^2)^{1/4}/f_1(n)\}^m]^{1/m} \quad (35)$$

where Pe_L and Ra_L are Pe_x and Ra_x at $x = L$. Equation (35) is obtained from equation (33) by knowing the average Nusselt number expressions for pure forced convection \overline{Nu}_f and pure free convection \overline{Nu}_n . They are found as

$$\overline{Nu}_f = 2f_1(n)Pe_L^{1/2} \quad (36)$$

and

$$\overline{Nu}_n = [4/(n+2)]f_2(n)Ra_L^{1/4}. \quad (37)$$

The average mixed convection Nusselt number from the prediction can be derived by finding the average heat transfer coefficient \bar{h} from the local Nusselt number expression given by equation (14). The end result is

$$\overline{Nu}Pe_L^{-1/2} = (1/n)\xi_{rL}^{-(1/2m)} \int_0^{\xi_{rL}} [\theta(\xi_r, 0)]^{-1} \xi_r^{(1-2m)/2n} d\xi_r \quad (38)$$

where $\xi_{rL} = \xi_r$ at $x = L$. As $n \rightarrow 0$, which is the limiting case of uniform heat flux, equation (38) reduces to the following expression

$$[\overline{Nu}Pe_L^{-1/2}]_{n=0} = 2[1/\theta(\xi_{rL}, 0)]. \quad (39)$$

An exponent value of $m = 3$ in equations (34) and (35) is found to correlate the predicted results very well. The maximum deviation between the correlated and the predicted mixed convection Nusselt numbers is found to be less than 5% for the range of $0 \leq n \leq 2$ over the entire regime of mixed convection.

CONCLUDING REMARKS

A nonsimilar solution for mixed convection from a heated horizontal impermeable flat plate in a saturated porous medium is reported for the case of variable surface heat flux. The entire mixed convection regime was covered by solving one system of equations for the forced convection dominated regime and another system of equations for the free convection dominated regime. A 5% rule was used to establish the regime where mixed convection becomes important for various surface heat flux distributions. Heat

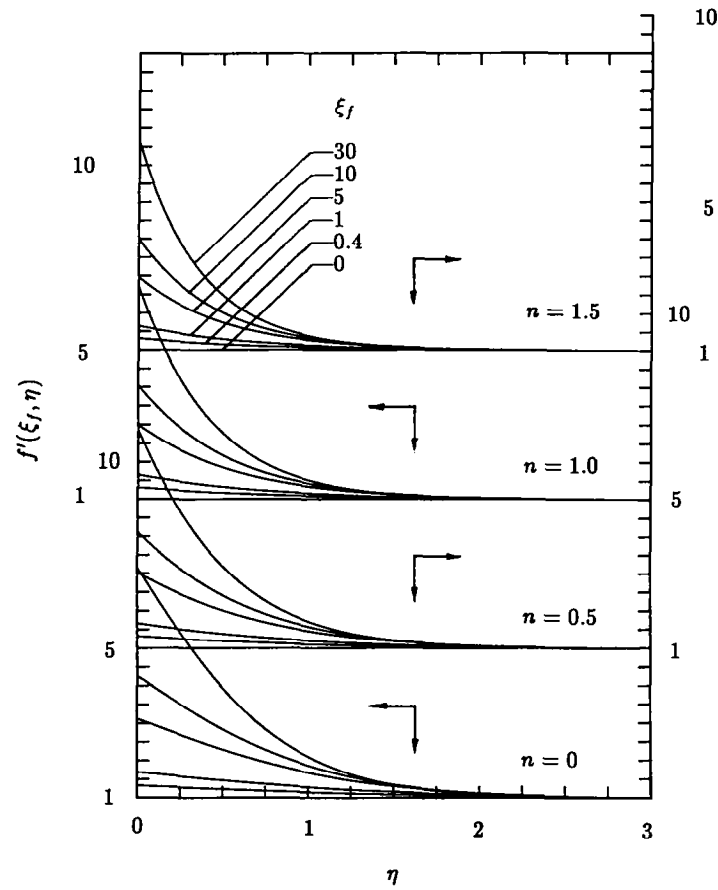


FIG. 2. Velocity profiles at selected values of ξ_f and n .

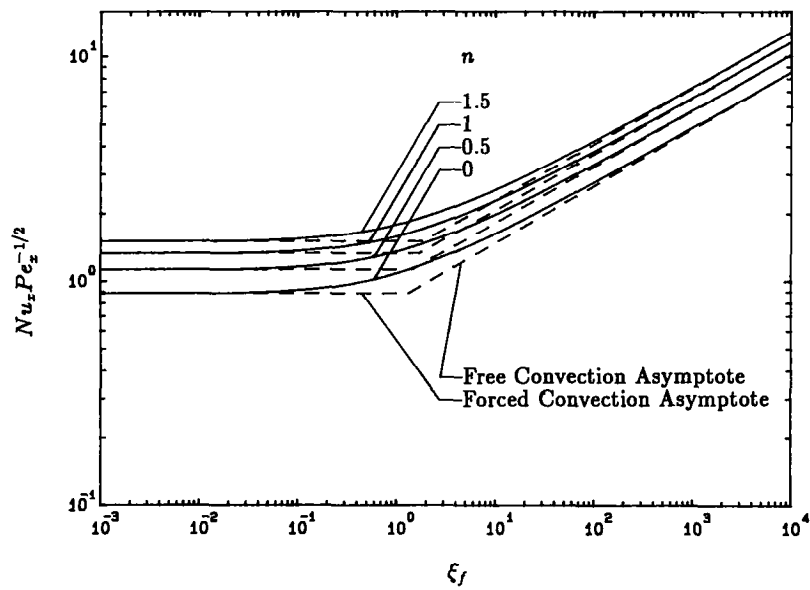


FIG. 3. Local Nusselt number variation for mixed convection with variable surface heat flux ($q_w = ax^n$).

Table 1. Domains of pure forced convection, mixed convection, and pure free convection

Exponent n	Range of $\zeta_r = Ra_x/Pe_x^2$ values for		
	Forced convection	Mixed convection	Free convection
0	0–0.17	0.17–14	14– ∞
0.5	0–0.15	0.15–31	31– ∞
1.0	0–0.13	0.13–50	50– ∞
1.5	0–0.12	0.12–84	84– ∞
2.0	0–0.11	0.11–115	115– ∞

transfer results from the 'local similarity' approximation is found to deviate from the nonsimilarity results by up to 10% at large values of the ζ_r parameter. However, it gives satisfactory results for small values of ζ_r and exact results for the limit cases of pure forced and pure free convection, as well as for mixed convection under the uniform surface heat flux condition. Simple and accurate correlations for the local and average Nusselt numbers were developed for the entire mixed convection regime.

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Table 2. Values of $1/\theta(\zeta_r, 0)$ and $1/\theta(\zeta_n, 0)$ at selected values of ζ_r and ζ_n for different values of n

ζ_r	$1/\theta(\zeta_r, 0)$				
	$n = 0$	$n = 0.5$	$n = 1.0$	$n = 1.5$	$n = 2$
0.0	0.886238E+00	0.112839E+01	0.132936E+01	0.150452E+01	0.166170E+01
0.1	0.916142E+00	0.116303E+01	0.136847E+01	0.154777E+01	0.170876E+01
0.2	0.942366E+00	0.119323E+01	0.140240E+01	0.158515E+01	0.174936E+01
0.3	0.965864E+00	0.121964E+01	0.143203E+01	0.161751E+01	0.178485E+01
0.4	0.987244E+00	0.124401E+01	0.145921E+01	0.164764E+01	0.181712E+01
0.5	0.100692E+01	0.126665E+01	0.148450E+01	0.167540E+01	0.184721E+01
0.6	0.102520E+01	0.128676E+01	0.150693E+01	0.170004E+01	0.187390E+01
0.7	0.104229E+01	0.130603E+01	0.152809E+01	0.172473E+01	0.190064E+01
0.8	0.105837E+01	0.132443E+01	0.154896E+01	0.174615E+01	0.192384E+01
0.9	0.107358E+01	0.134222E+01	0.156820E+01	0.176717E+01	0.194652E+01
1.0	0.108802E+01	0.135818E+01	0.158658E+01	0.178742E+01	0.196853E+01
ζ_n	$1/\theta(\zeta_n, 0)$				
	$n = 0$	$n = 0.5$	$n = 1.0$	$n = 1.5$	$n = 2$
1.0	0.108802E+01	0.135818E+01	0.158658E+01	0.178742E+01	0.196853E+01
0.9	0.106190E+01	0.132265E+01	0.154231E+01	0.173657E+01	0.191286E+01
0.8	0.103607E+01	0.128650E+01	0.149845E+01	0.168575E+01	0.185490E+01
0.7	0.101066E+01	0.125153E+01	0.145453E+01	0.163557E+01	0.179756E+01
0.6	0.985795E+00	0.121534E+01	0.141106E+01	0.158444E+01	0.174035E+01
0.5	0.961626E+00	0.117898E+01	0.136617E+01	0.153293E+01	0.168296E+01
0.4	0.938332E+00	0.114607E+01	0.132506E+01	0.148275E+01	0.162588E+01
0.3	0.916118E+00	0.111262E+01	0.128152E+01	0.143244E+01	0.156923E+01
0.2	0.895234E+00	0.108126E+01	0.124076E+01	0.138303E+01	0.151266E+01
0.1	0.876002E+00	0.105167E+01	0.120157E+01	0.133566E+01	0.145745E+01
0.0	0.858906E+00	0.102479E+01	0.116507E+01	0.128970E+01	0.140331E+01

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